ASTR 1010: Math Review Session

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1 Algebraic Manipulation

1.1 Definitions

- **variable** - a number that can vary; our "unknown"
- **coefficient** - a constant multiplier; the "number in front"
- **constant** - a known number
- **term** - numbers and variables multiplied together
- **operator** - symbol showing an operation (what to do with numbers)
- **expression** - a group of terms
- **equation** - a statement that two or more expressions are equal

Question 1

Label the equation below with the words defined above:

$$3x + 2y + 5 = 11.$$

1.2 Fundamental Ideas

- Collecting like terms:
  What are like terms?
  e.g. $3x + 2x = 5x$ but $3x + 2y$ cannot be simplified further.
- Operating on both sides of an equation to simplify:
  subject - the quantity of interest; made alone, usually on the left-hand side (LHS)

  1. Identify the term(s) with the variable of interest.
  2. Perform operations so that the term(s) with the variable of interest is on the LHS and the other terms are on the right-hand side (RHS).
  3. Perform operations to make the variable of interest the subject.

Question 2

Let's manipulate $3x + 2y + 5 = 11$ to make x the subject.

Question 3

The equation of state for a van der Waals gas is given by:

$$\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT.$$

Let's solve for p.

Work on Math Review Worksheet Section 1

2 Powers and Roots

2.1 Fundamental Ideas

- Building an intuition for exponents:
  - Positive exponents:
  $$1 = a^0 \to a = a^1 \to a \times a = a^2 \to a \times a \times a = a^3 \to \cdots$$
  - Negative exponents:
  $$1 = a^0 \to \frac{1}{a} = a^{-1} \to \frac{1}{a^2} = a^{-2} \to \frac{1}{a^3} = a^{-3} \to \cdots$$
  - Roots and fractional exponents:
  $$1 = a^0 \to \sqrt{a} = a^{1/2} \to a = a^1$$
  $$1 = a^0 \to \sqrt[3]{a} = a^{1/3} \to (\sqrt[3]{a})^2 = a^{2/3} \to a = a^1$$
  etc.
- Laws of Exponents:
  - When you multiply exponents, you add them.
  e.g. $a^2 \times a^3 = a \times a \times a \times a \times a = a^5$
  - When you divide exponents, you subtract them.
  e.g. $\frac{b^4}{b^2} = \frac{b \times b \times b \times b}{b \times b} = b^2$
  - When you take the power of an exponent, you multiply them.
  e.g. $(c^3)^2 = c^3 \times c^3 = c^6$

Question 3

Simplify the following:

(a) $w^4 \times w^3$

(b) $x^2 \times \sqrt{x}$

(c) $\frac{y^4 \times \sqrt[3]{y}}{y^{2/3}}$

(d) $\frac{(z^{3/2})^2}{(\sqrt{z})^3}$

Work on Math Review Worksheet Section 2

3 Scientific Notation

3.1 Fundamental Ideas

- Basic concept: writing numbers that are too large or too small using powers of 10. The number in front should be $1 \leq a < 10$.
  e.g. $5,000,000 = 5 \times 10^6$
  e.g. $0.00007 = 7 \times 10^{-5}$

- Multiplying and dividing numbers represented in scientific notation.

  – Multiplying:
  $$(a \times 10^n) \times (b \times 10^m) = (a \times b) \times 10^{n+m}$$

  – Dividing:
  $$\frac{a \times 10^n}{b \times 10^m} = \left(\frac{a}{b}\right) \times 10^{n-m}$$

Question 4

(a) Write the speed of light ($c = 299,792,458$ m/s) in scientific notation.

(b) The Sun is roughly 1.496×10^{11} m away from Earth. How long does it take for light to travel from the Sun to Earth (in seconds)?

Work on Math Review Worksheet Section 3

4 Unit Conversion and Prefixes

4.1 Fundamental Ideas

- The three most fundamental physical quantities are length, mass, and time. The base units for these three are usually meters, kilograms, and seconds (the MKS system) or centimeters, grams, and seconds (the CGS system). Either way, people use alternative units, so unit conversion is a necessary skill.

- ALWAYS carry your units when you perform calculations. It gives you clarity and a way to double-check you answer.
  e.g. $700 \times 10^{10}\ \text{s} = 700 \times 10^{10}\ \text{s} \times \frac{1\ \text{yr}}{3.16 \times 10^7\ \text{s}} \approx 200,000\ \text{yr}$

Unit Prefix	Prefix Symbol	Power of 10
Milli-	m	10^{-3}
Centi-	c	10^{-2}
Kilo-	k	10^3
Mega-	M	10^6
Giga-	G	10^9
Tera-	T	10^{12}

Question 5

(a) Convert 4×10^9 m into Mm.

(b) Convert the speed of light ($c \approx 3 \times 10^8$ m/s) into ft/yr.

(c) Convert $5 \times 10^{-3}\ \text{cm}^2/\text{s}^2$ into km^2/ms^2.

Work on Math Review Worksheet Section 4

5 Significant Figures

5.1 Fundamental Ideas

- Digits that are NOT significant:
  - Leading zeros
    e.g. 0.00013 has 2 significant figures
  - Trailing zeros before decimal point
    e.g. 7200 has 2 significant figures
- Significant figures are easier to count when the number is expressed in scientific notation.
  e.g. $0.00013 = 1.3 \times 10^{-4}$, $7200 = 7.2 \times 10^{3}$
- Usually, you give your answer to the least number of significant figures used in your calculation, but when in doubt report your answer to 2 or 3 significant figures.
  e.g. $10.2/2 = 5$ where as $10.2/2.0 = 5.1$.

Question 6

(a) Represent the speed of light ($c = 299,792,458$ m/s) as a 3 significant figure number.

(b) Represent the radius of the Sun ($R = 6.95 \times 10^{5}$ km) as a 1 significant figure number.

(c) Report the number of seconds in a year to the appropriate number of significant figures.

Work on Math Review Worksheet Section 5

6 Areas and Volumes

6.1 Fundamental Ideas

- Working with different shapes is a part of studying astronomy, so we need to learn how to work with areas and volumes.
- Specifically, circles and spheres occur A LOT in astronomy. Some key equations include:
  - Circumference (or perimeter) of circle: $P = 2\pi r$;
  - Area of circle: $A = \pi r^{2}$;
  - Surface area of sphere: $A = 4\pi R^{2}$;
  - Volume of sphere: $V = \frac{4}{3}\pi R^{3}$.

Question 7

(a) Calculate the light collecting area of our 20" telescope. Report your answer in cm^{2}. (Note: 20" is the *diameter* of our telescope.)

(b) Calculate the volume of our Sun given the fact that its radius is roughly $R = 6.95 \times 10^{5}$ km. Report your answer in m^{3}.

Work on Math Review Worksheet Section 6