Math Review Worksheet

January 21, 2020

1 Algebraic Manipulation

Problem 1

Manipulate the following equations to make x the subject:

(a) $2x + 6y = 12$

(b) $5x - \frac{3}{2}y = -2x + 15$

(c) $3x - 2y + 5z - 10 = 13 + x + 4y$

(d) $\frac{4x-2y}{3} = 5$

(e) $\frac{7-x}{10y-3x} = 6$

(f) $\frac{6-y}{3} = \frac{4}{2x+9}$

(g) $\sqrt{x + y - 2} + 5y = 10$

(h) $(3x - 8 + 2y)^3 = 8 + y$

(i) $\frac{\sqrt[3]{2y-x}}{5} = y + 2$

Problem 2

Make x the subject of the equation:

$$z = \frac{x - \mu}{\sigma}.$$

Problem 3

The equation to convert from degrees in Fahrenheit to Kelvin (base unit for temperature) is:

$$T_K = \frac{T_F - 32}{1.8} + 273.15.$$

Rearrange the equation to covert from Kelvin to degrees Fahrenheit.

Problem 4

Newton's law of gravitation says that:

$$F = G\frac{m_1 m_2}{r^2}.$$

(a) Solve for m_1.

(b) Make r the subject of this equation.

Problem 5

Here is a version of Kepler's Third Law:

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3.$$

(a) Make m_1 the subject of this equation.

(b) Solve for a.

Problem 6

The Lorentz factor is calculated by the equation:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}.$$

Given a certain Lorentz factor, how would you calculate the speed v?

2 Powers and Roots

Problem 7

Simplify the following:

(a) $a \times a^6$

(b) $w^6 \times w^{-2}$

(c) $b^3 \times \sqrt[5]{b}$

(d) $\sqrt{x} \times \sqrt[3]{x}$

(e) $\frac{c^{-2} \times (\sqrt{c})^3}{c^{-2/3}}$

(f) $\frac{y \times \sqrt[4]{y}}{y^2 \times y^{-3}}$

(g) $\frac{(d^{3/2})^{-4}}{(\sqrt[3]{d})^4}$

(h) $\frac{(1/z^3)^{-4/3} \times \sqrt[3]{z}}{\sqrt{z^{-2}}}$

3 Scientific Notation

Problem 8

Write the following in scientific notation:

(a) the fine structure constant ($\alpha = 0.007297352533$);

(b) the Rydberg constant ($R = 10973731.568549\ \mathrm{m}^{-1}$);

(c) the Stefan-Boltzmann constant ($\sigma = 0.00000005670400\ \mathrm{Wm}^{-2}\mathrm{K}^{-4}$);

(d) the constant of gravitation ($G = 0.00000000006673\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$).

Problem 9

Mars, at its closest, is roughly 128 million miles away from Earth. How fast do we have to be traveling (in mph) for us to get there in a year?

Problem 10

Let's revisit Newton's law of gravitation which gives the force of gravity between two masses m_1 and m_2 a distance r apart:

$$F = G\frac{m_1 m_2}{r^2}.$$

The constant of gravitation G is given in Problem 8 (d). Calculate the force of gravity between the Sun and Earth given that the mass of the Sun is 1.989×10^{30} kg, the mass of Earth is 5.972×10^{24} kg, and the distance between the Sun and Earth is on average around 150 billion meters.

4 Unit Conversion and Prefixes

Problem 11

(a) Convert 7×10^{-4} m into mm.

(b) Convert 3.4×10^{9} kg into Gg.

(c) Convert the Rydberg constant ($R = 10973731.568549$ m^{-1}) into cm^{-1}.

(d) Convert the constant of gravitation ($G = 0.00000000006673$ m^{3}kg^{-1}s^{-2}) into km^3 g^{-1} day^{-2}.

5 Significant Figures

Problem 12

(a) Represent the fine structure constant ($\alpha = 0.007297352533$) as a 3 significant figure number.

(b) Represent the Rydberg constant ($R = 10973731.568549$ m^{-1}) as a 2 significant figure number.

(c) Represent the Stefan-Boltzmann constant ($\sigma = 0.00000005670400$ Wm^{-2}K^{-4}) as a 4 significant figure number.

(d) Represent the constant of gravitation ($G = 0.00000000006673$ m^{3}kg^{-1}s^{-2}) as a 1 significant figure number.

Problem 13

Calculate your age in seconds to the appropriate number of significant figures.

Problem 14

Calculate your height in mm to the appropriate number of significant figures.

6 Areas and Volumes

Question 7

(a) Calculate the light collecting area of the Keck telescope that is 32.8 feet across (in diameter). Report your answer in m^2.

(b) Calculate the volume of Jupiter given the fact that its radius is around $R = 43,441$ miles. Report your answer in km^3.

(c) Given that the circumference of your wrist is around 5 inches, what is the radius of your wrist (roughly)?

(d) The Sun gives off around 3.846×10^{26} J of energy per second. Given that the radius of the Sun is around 432,690 miles, roughly how much energy is radiated by a meter by meter square on the Sun?