Young's Double-Slit Experiment

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This paper reports the results of analyzing the double-slit interference and single-slit diffraction patterns for a Helium-Neon laser and Mercury's green spectral line. We use Helium-Neon laser of known wavelength to determine the slit separation and slit width. Using these experimental values of the slit separation and slit width and treating the double-slit interference and single-slit diffraction phenomena individually, we observe the interference pattern created by Mercury's green spectral line to obtain predictions of its wavelength. The experimental value of the wavelength is compared to the true value to assess the experiment's accuracy.

I. INTRODUCTION

I.1. Double-Slit Interference

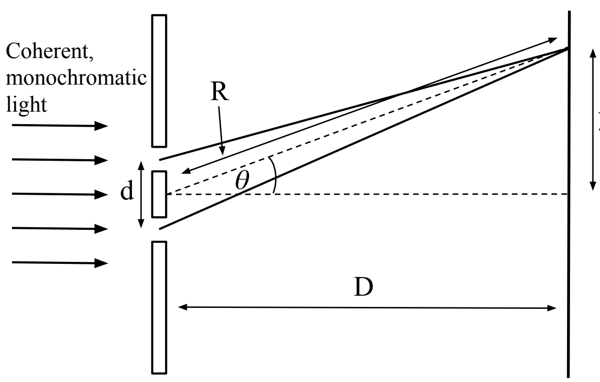

FIG. 1. This is a schematic diagram of the general geometry of Young's double-slit experiment.

In 1803, Thomas Young performed an experiment which posed a major challenge to the particle theory of light [1]. This experiment, now known as Young's double-slit experiment, showed that coherent, monochromatic light passing through two slits produces an interference pattern on a distant screen, a clear indication that light exhibits wave phenomena [1]. A schematic diagram of the general geometry of Young's double-slit experiment is shown in Figure 1, where d is the distance between the slits, D is the distance between the pair of slits and the screen, R is the approximate distance the light rays travel, y is the distance between the center and the point of interest, and θ is the approximate angle between the light rays and the normal to the screen, as shown.

In order to determine the interference of the two light rays at the point of interest, we make use of the geometry shown in Figure 2, which is effectively a close-up of the geometry presented in Figure 1 near the two slits. We note that if the screen is distant such that $D >> d$, the two light rays are essentially parallel. Thus, we can use similar triangles to see that the path difference between the two light rays, ε, is approximately the side opposite θ in the smaller triangle shown in Figure 2. Therefore,

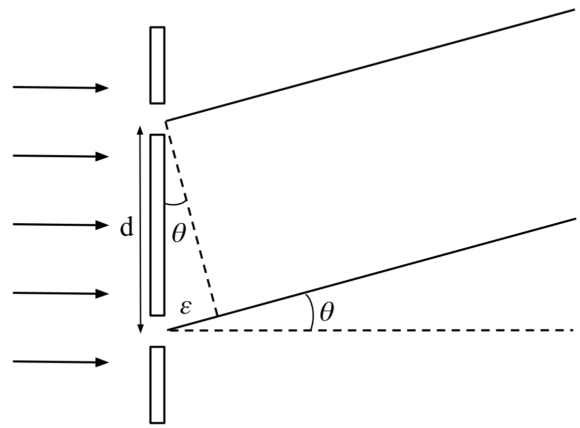

FIG. 2. This is a schematic diagram of the geometry of Young's double-slit experiment near the two slits.

$$\varepsilon = d\sin(\theta).$$

For the n^{th} constructive interference away from the central maxima, we would like this path difference to be exactly n wavelengths, so we obtain:

$$n\lambda = d\sin(\theta_n), \quad (1)$$

where λ is the wavelength of light and we have labeled the particular angle at which the n^{th} constructive interference occurs as θ_n.

Hence, for neighboring bright fringes exhibiting the m^{th} and $(m+1)^{\text{st}}$ constructive interference, we have:

$$m\lambda = d\sin(\theta_m), \quad (2)$$

$$(m+1)\lambda = d\sin(\theta_{m+1}). \quad (3)$$

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Subtracting Equation 2 from Equation 3 and rearranging, we obtain:

$$\frac{\lambda}{d} = \sin(\theta_{m+1}) - \sin(\theta_m) \approx \theta_{m+1} - \theta_m, \tag{4}$$

where we have used the small-angle approximation for our final step.

Now, from Figure 1, we see that $y = R\sin(\theta)$, so for neighboring slits at m and $m+1$, the spacing between the bright fringes, δ, is going to be given by:

$$\begin{aligned}\delta &= y_{m+1} - y_m \\ &= R\sin(\theta_{m+1}) - R\sin(\theta_m) \\ &\approx R(\theta_{m+1} - \theta_m) \\ &\approx D(\theta_{m+1} - \theta_m),\end{aligned}$$

where we have used the approximation $R \approx D$ for our final step. Finally, this result combined with Equation 4 yields the equation central to our experiment:

$$\delta \approx \frac{D\lambda}{d}. \tag{5}$$

[2]

In this experiment, we will use Equation 5 to first determine an experimental value for the distance between the two slits d with a Helium-Neon laser of known wavelength. Then, we will use the value of d obtained to find the wavelength of the Mercury's spectrum's green line.

I.2. Single-Slit Diffraction

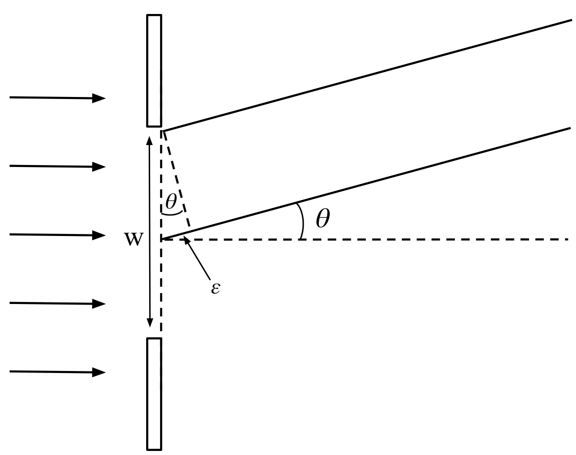

FIG. 3. This is a schematic diagram of the geometry of the single-slit diffraction phenomenon.

In this experiment, we will also take into account the phenomenon of single-slit diffraction to obtain an experimental value for the slit width, w, for each slit and a second prediction for the wavelength of Mercury's green spectrum line. A schematic diagram of the single-slit diffraction phenomenon is shown in Figure 3. In order for destructive interference to occur in this situation, we would want each of the rays that are initially $w/2$ apart along the slit to destructively interfere, since all the light rays passing through the slit can be effectively paired up to destructively interfere in this fashion, under the assumption that all the light rays are parallel [3]. Using a similar argument as the double-slit set-up, we see that the path difference, ε, between a pair of light rays initially $w/2$ apart is:

$$\varepsilon = \left(\frac{w}{2}\right)\sin(\theta).$$

For the first minimum, we would like this path difference to be equal to half the wavelength of the light so that they interfere destructively. Hence,

$$\begin{aligned}\frac{\lambda}{2} &= \left(\frac{w}{2}\right)\sin(\theta) \\ \implies \lambda &= w\sin(\theta)\end{aligned} \tag{6}$$

We can follow the same argument to derive that all dark fringes can be located by:

$$n\lambda = w\sin(\theta_n), \tag{7}$$

[3] where $n = \pm 1, \pm 2, \ldots$. Notice that, conveniently, this is essentially identical to the result we obtained for the double-slit experiment, Equation 1. Therefore, we can follow the same geometric arguments we made in the double-slit case to obtain that the fringe spacing, δ, is given by

$$\delta \approx \frac{D\lambda}{w}. \tag{8}$$

In this experiment, we will observe both double-slit interference and single-slit diffraction. Since $d > w$ in our double-slit set-up, according to Equations 1 and 7, we will see a pattern similar to the one shown in Figure 4, where we have a more closely spaced double-slit interference pattern modulated by the single-slit envelope caused by diffraction. Note that we can find the number of bright, sharp fringes we expect to find in a given broad peak by taking the ratio between Equations 5 and 8:

$$\begin{aligned}\text{Number of fringes} &= \frac{D\lambda/w}{D\lambda/d} \\ &= \frac{d}{w}.\end{aligned}$$

II. PROCEDURE

II.1. Determining d and w Using the Helium-Neon Laser

The experimental set-up for using the Helium-Neon laser with our double-slit experiment is shown in Figure

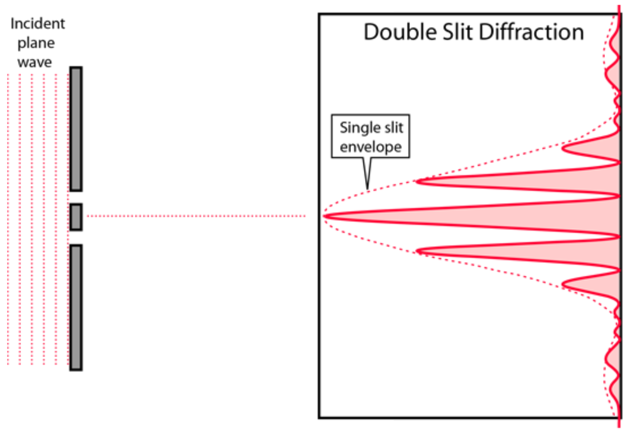

FIG. 4. This figure shows the resultant interference pattern we should expect to see as a result of the combination of the double-slit interference and single-slit diffraction phenomena.

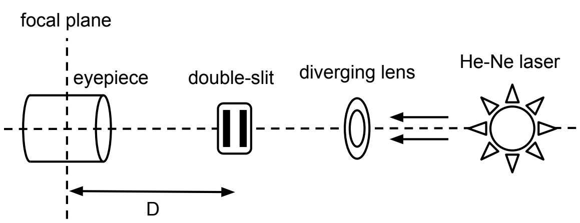

FIG. 5. This is a schematic diagram of our experimental set-up for observing the interference pattern of Helium-Neon laser.

5. The light is first shined through a diverging lens to decrease its intensity before encountering the two slits, and the interference pattern is observed using an eyepiece with an embedded micrometer that allows us to measure the fringe spacing, δ. For each D, we make five measurements for δ at different neighboring pairs and average to reduce uncertainty, and we do this for a range of D's to determine the spacing between the slits, d, using Equation 5. Similarly, we measure the spacing between a pair of minima for the single-slit envelope at various D's and use Equation 8 to determine the slit width, w.

II.2. Measuring the Wavelength of Mercury's Green Spectrum Line

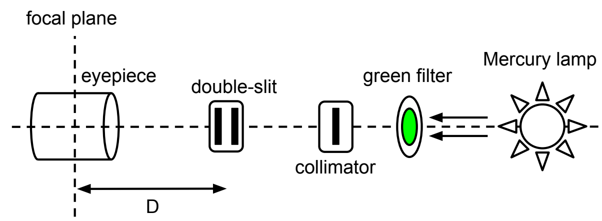

FIG. 6. This is a schematic diagram of our experimental set-up for observing the interference pattern of Mercury's green spectral line.

The experimental set-up for using the Mercury lamp is shown in Figure 10. The light is first passed through a green filter to pick out the green spectral line of Mercury. Then, the rays along the line connecting the double-slit and the eyepiece are picked out by the single-slit collimator for alignment. The outgoing light is then passed through the two slits and the interference pattern observed by the eyepiece just as with the Helium-Neon laser. Again, we measure the fringe spacing, δ, for various D's taking five measurements of δ for each D, but this time, we use the experimental value of d we obtained from our first data set for the Helium-Neon laser to compute the wavelength of the Mercury's green line. Similarly, we measure the spacing between two of the minima for the single-slit envelope for each D and compute a second value of the wavelength of the Mercury's green line based on single-slit diffraction. We report our final result based on the results of the two values treating the double-slit interference and single-slit diffraction, respectively.

III. RESULTS

III.1. Preliminary Data and Measurements

The wavelength of the Helium-Neon laser used is (632.8 ± 0.1)nm, and the wavelength of Mercury's green spectral line is (546.2 ± 0.1)nm [2]. We will use the known wavelength of the Helium-Neon laser to obtain an experimental value for the slit separation, d, and use this experimental value to predict the wavelength of Mercury's green spectral line. We compare our experimental prediction with the true value to assess the accuracy of our experiment.

To measure the fringe spacing using the embedded micrometer, we first take an initial reading on the micrometer when the two crosshairs overlap, and take the difference between the readings when the two crosshairs are aligned with our fringes of interest. The initial micrometer reading when the two crosshairs overlapped was (5.215 ± 0.005)mm.

III.2. Measuring the Separation of the Slits Using Double-Slit Interference of Helium-Neon Laser

We measured the fringe separation, δ, for each D, the distance between the slits and the eyepiece, for D varying between (0.263 ± 0.001)m and (0.663 ± 0.001)m using the experimental set-up show in Figure 5. δ was measured using the embedded micrometer of the eyepiece and D was measured using a ruler running along the double-slit experimental set-up. We took five measurements of δ for individual neighbor pairs of fringes for each D and treated these statistically. The average value of δ was plotted against D and is shown with the linear fit and its equation in Figure 7. We report the slope of the linear fit to be $(1.07 \pm 0.01) \times 10^{-3}$ (unitless). According to Equation 5, the slope of this linear fit, m, corresponds to λ/d, so we use the known wavelength of the Helium-Neon laser and the slope of the linear fit to obtain an experimental value of d as follows:

$$\begin{aligned} d &= \frac{\lambda}{m} \\ &= \frac{(632.8 \pm 0.1) \times 10^{-9}\text{m}}{(1.07 \pm 0.01) \times 10^{-3}} \\ &\approx (5.91 \pm 0.06) \times 10^{-4}\text{m}. \end{aligned}$$

III.3. Measuring the Slit Width Using Single-Slit Diffraction of Helium-Neon Laser

Following a similar process as Section III.2 with the double-slit interference of the Helium-Neon laser, we now measure the separation between neighboring dark fringes of the single-slit diffraction pattern for varying D's. We use D's ranging in value from (0.263 ± 0.001)m to (0.563 ± 0.001)m. Since these were much broader than the double-slit interference pattern, we were only able to take a single measurement of the spacing for each D. The fringe spacing for the single-slit diffraction pattern was plotted against D and is shown with the linear fit and its equation in Figure 8. We report the slope of this linear fit to be $(5.12 \pm 0.06) \times 10^{-3}$ (unitless). According to Equation 8, the slope of this linear fit corresponds to λ/w, so we obtain an experimental value of w:

$$\begin{aligned} w &= \frac{\lambda}{m} \\ &= \frac{(632.8 \pm 0.1) \times 10^{-9}\text{m}}{(5.12 \pm 0.06) \times 10^{-3}} \\ &\approx (1.24 \pm 0.01) \times 10^{-4}\text{m}. \end{aligned}$$

III.4. Determining the Wavelength of Mercury's Green Spectral Line Using Double-Slit Interference

Now, we perform the same experiment as in Section III.2 but using the Mercury lamp and the experimental set-up shown in Figure 10. We use D's ranging in value from (0.263 ± 0.001)m to (0.513 ± 0.001)m. The average value of δ was plotted against D and is shown with the linear fit and its equation in Figure 9. We report the slope of the linear fit to be $(9.9 \pm 0.8) \times 10^{-4}$ (unitless). Since this slope, m corresponds to λ/d, we can now use the experimental value of d determined in Section III.2 to obtain a prediction of the wavelength of Mercury's green spectral line.

$$\begin{aligned} \lambda &= md \\ &= (9.9 \pm 0.8) \times 10^{-4} \times (5.91 \pm 0.06) \times 10^{-4}\text{m} \\ &\approx (5.9 \pm 0.5) \times 10^{-7}\text{m}. \end{aligned}$$

III.5. Determining the Wavelength of Mercury's Green Spectral Line Using Single-Slit Diffraction

We perform the same experiment as in Section III.3 but using the Mercury lamp and the set-up shown in Figure 10. We use D's ranging in value from (0.263 ± 0.001)m to (0.563 ± 0.001)m. The separation between two dark fringes of the single-slit diffraction pattern was plotted against D and is shown with the linear fit and its equation in Figure **??**. We report the slope of the linear fit to be $(4.32 \pm 0.08) \times 10^{-3}$ (unitless). Since this slope, m corresponds to λ/w, we use the experimental value of w determined in Section III.3 to obtain another prediction of the wavelength of Mercury's green spectral line.

$$\begin{aligned} \lambda &= mw \\ &= (4.32 \pm 0.08) \times 10^{-3} \times (1.24 \pm 0.01) \times 10^{-4}\text{m} \\ &\approx (5.3 \pm 0.1) \times 10^{-7}\text{m}. \end{aligned}$$

III.6. Final Results

We report the slit separation, d, and the slit width, w, to be $(5.91 \pm 0.06) \times 10^{-4}$m and $(1.24 \pm 0.01) \times 10^{-4}$m respectively. From the two predictions of the wavelength

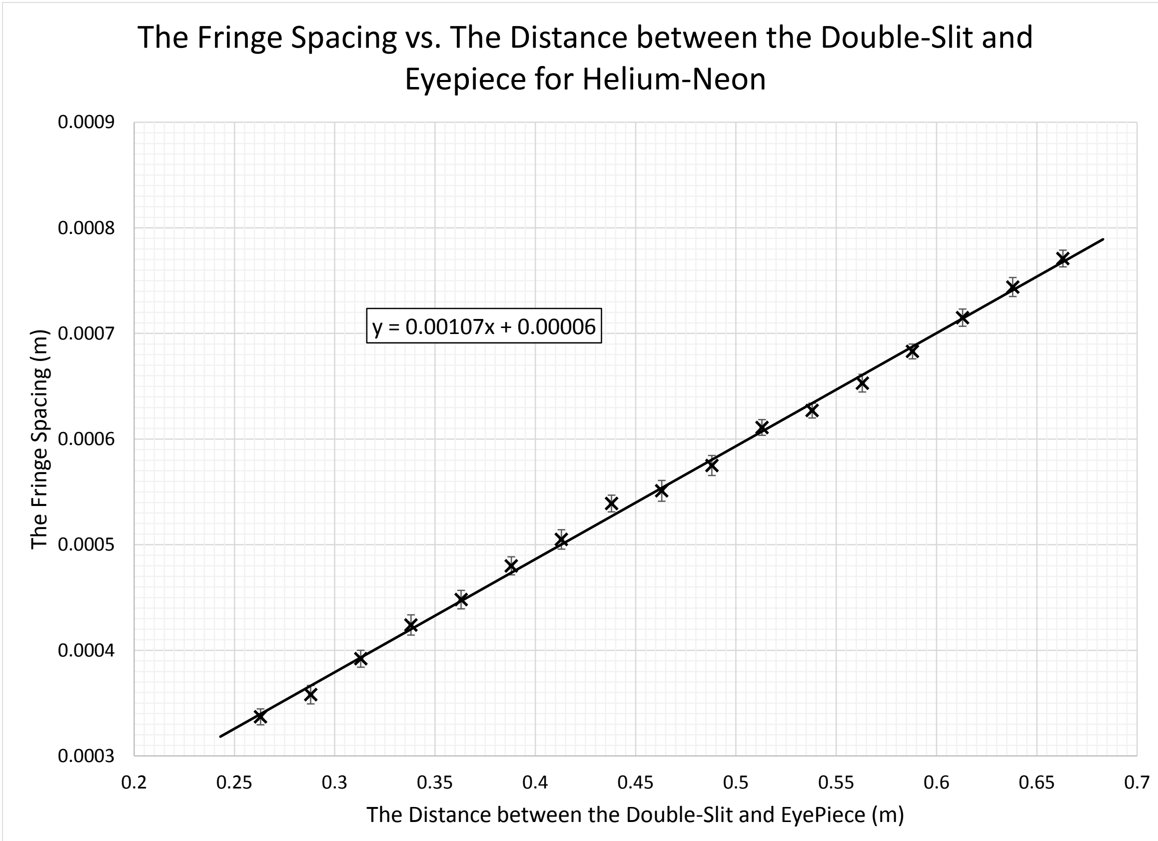

FIG. 7. This a plot of the fringe spacing for the double-slit interference pattern against the distance between the two slits and the eyepiece together with its linear fit and equation for Helium-Neon.

of Mercury's green spectral line obtained using double-slit interference and single-slit diffraction, we obtain our final experimental value of the wavelength of Mercury's green spectral line by computing an average weighted by the inverse square of their uncertainties. Hence, we report the final result for the wavelength of Mercury's green spectral line to be:

$$\lambda = (5.3 \pm 0.3) \times 10^{-7}\text{m}.$$

IV. ERROR ANALYSIS

IV.1. Uncertainty Propagation

We show a sample uncertainty propagation calculation for the first value of D in the Helium-Neon laser double-interference data set. For $D = (0.263 \pm 0.001)$m, we obtained micrometer readings of 5.550mm, 5.555mm, 5.545mm, 5.550mm, and 5.560mm each with an uncertainty of 0.005mm. These values give us an average reading of 5.552mm with standard deviation approximately equal to 0.0057mm. From this data, we compute the standard error and use that as our uncertainty value for the average:

$$\begin{aligned}\text{Standard Error} &= \frac{s}{\sqrt{N}} \\ &= \frac{0.0057\text{mm}}{\sqrt{5}} \\ &\approx 0.003\text{mm}.\end{aligned}$$

From the average value of the micrometer reading, the initial value of (5.215 ± 0.005)mm is subtracted, so a simple addition of uncertainties gives us the following δ and its uncertainty:

$$\delta = (3.37 \pm 0.08) \times 10^{-4}\text{m}.$$

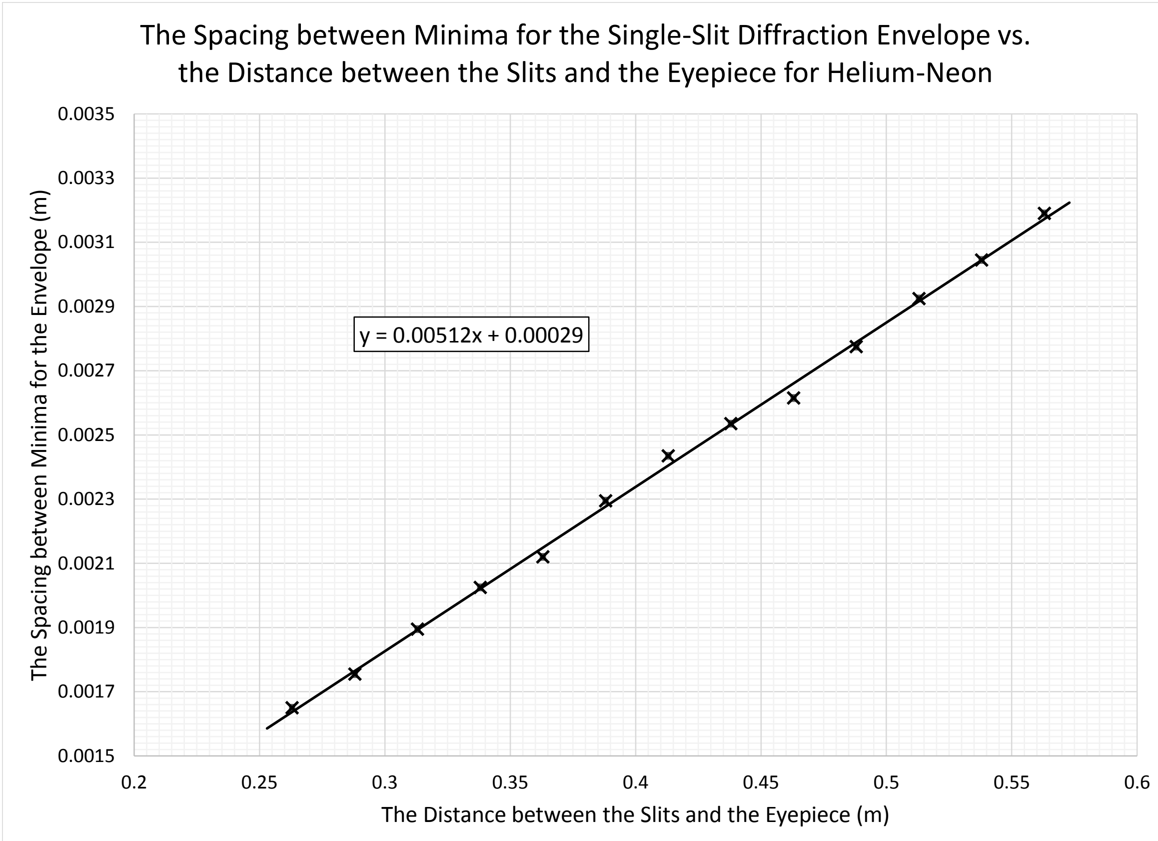

FIG. 8. This a plot of the fringe spacing for the single-slit diffraction pattern against the distance between the two slits and the eyepiece together with its linear fit and equation for Helium-Neon.

The uncertainties for each datum on our plots were computed in a similar manner and the error bars are shown in the figures.

IV.2. Linear Regression

Following Hughes & Hase, we calculate the uncertainty in our slope of linear fit, and thus the uncertainty of our experimental values of the Verdet constant using:

$$\alpha_{CU} = \sqrt{\frac{1}{N-2}\sum_i (y_i - mx_i - c)^2}, \text{ and}$$

$$\alpha_m = \alpha_{CU}\sqrt{\frac{N}{\Delta}},$$

where $\Delta = N\sum_i x_i^2 - \left(\sum_i x_i\right)^2$ [4].

IV.3. Further Uncertainty Propagation

Here, we show a sample uncertainty propagation to obtain our value and uncertainty for d in our experiment observing the double-slit interference of Helium-Neon. Since $d = \lambda/m$ where λ is the wavelength of the Helium-Neon laser light and m is the slope of our linear fit, we perform the addition of uncertainties in quadrature, following Hughes & Hase,

$$\frac{\alpha_d}{d} = \sqrt{\left(\frac{\alpha_\lambda}{\lambda}\right)^2 + \left(\frac{\alpha_m}{m}\right)^2}$$

This results in our final reporting of d as $(5.91 \pm 0.06) \times 10^{-4}$m.

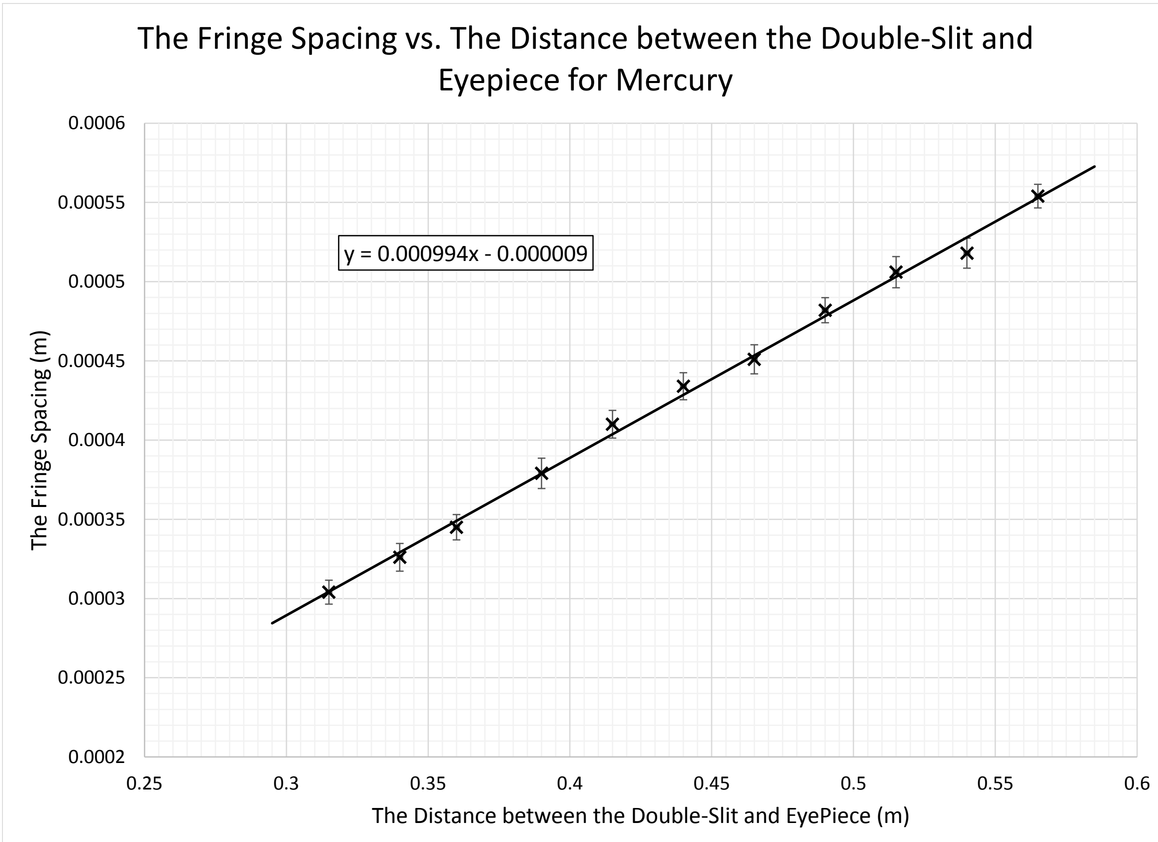

FIG. 9. This a plot of the fringe spacing for the double-slit interference pattern against the distance between the two slits and the eyepiece together with its linear fit and equation for Mercury.

IV.4. Percentage Error and Accuracy

Comparing our experimental value of the wavelength of Mercury's green spectral line to the true value of (546.2 ± 0.1)nm, we obtain the percentage error:

$$\begin{aligned}\%_e &= \frac{\theta_{\text{exp}} - \theta_{\text{the}}}{\theta_{\text{the}}} \times 100\% \\ &= \frac{5.3 \times 10^{-7}\text{m} - 5.462 \times 10^{-7}\text{m}}{5.462 \times 10^{-7}\text{m}} \times 100\% \\ &= -2.97\%.\end{aligned}$$

Therefore, our experimental value exhibits a high degree of accuracy as we were able to predict the wavelength of Mercury's green spectral line to within a 3% error. The individual predictions made by treating the double-slit interference and single-slit diffraction individually were $(5.9 \pm 0.5) \times 10^{-7}$m and $(5.3 \pm 0.1) \times 10^{-7}$m, respectively. The true value of the wavelength of Mercury's green line falls within our uncertainty range for the double-slit, but falls just out of our uncertainty range for the single-slit, suggesting the presence of some systematic error in our observations of single-slit diffraction.

V. CONCLUSION

In this experiment, we successfully measured the slit separation, d, of our double-slit set-up and the slit width, w, by treating the double-slit interference phenomenon and single-slit diffraction phenomenon separately. By measuring both the sharper fringe spacing corresponding to the double-slit interference pattern and the wider envelope's fringe spacing corresponding to the single-slit diffraction pattern at various slit-to-eyepiece distances for a Helium-Neon laser of known wavelength, we applied Equations 5 and 8 to obtain experimental values of d and w. Then, we made the same observations for the

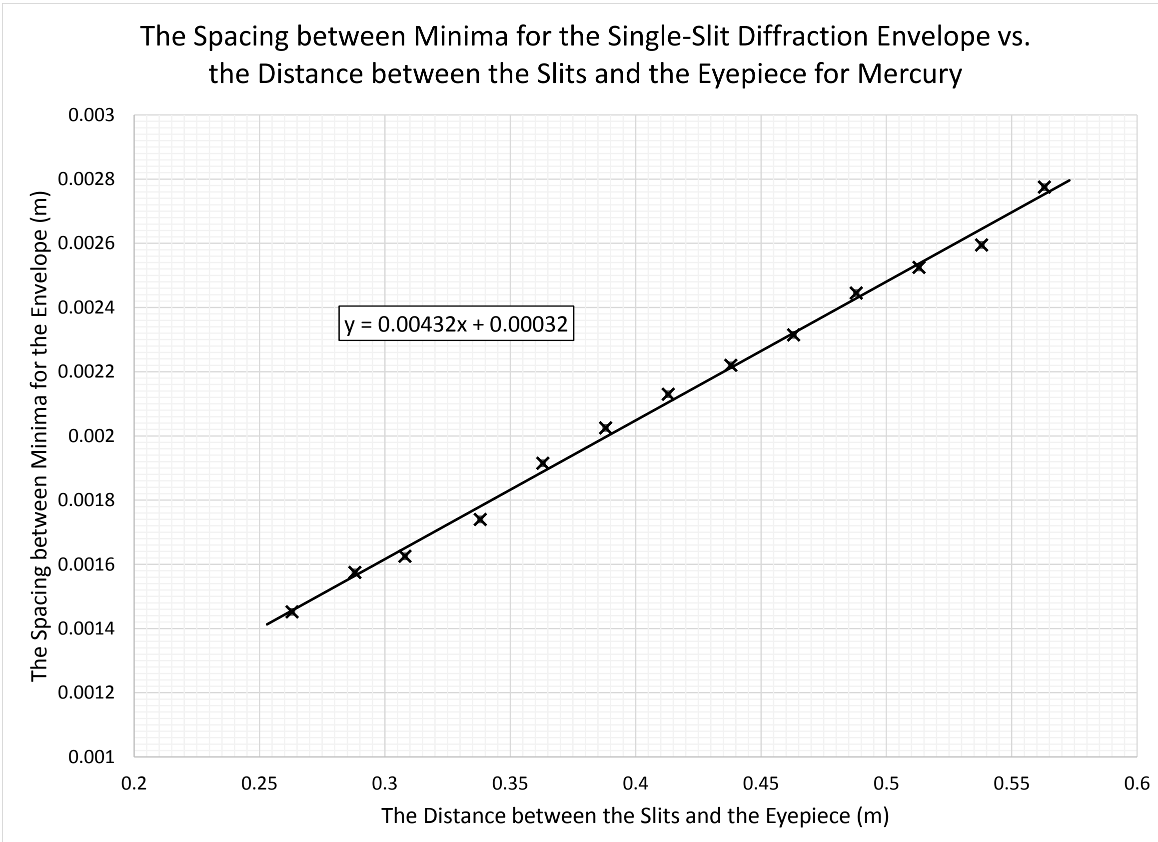

FIG. 10. This a plot of the fringe spacing for the single-slit diffraction pattern against the distance between the two slits and the eyepiece together with its linear fit and equation for Mercury.

interference and diffraction of Mercury's green spectral line, and applied the same equations and the values of d and w determined to compute predictions for the wavelength of Mercury's green line. The experimental value was compared to the true value of the wavelength of Mercury's green spectral line to evaluate the accuracy of our experiment.

From the first part of our experiment, we obtained the slit separation and the slit width to be:

$$d = (5.91 \pm 0.06) \times 10^{-4}\text{m},$$

$$w = (1.24 \pm 0.01) \times 10^{-4}\text{m}.$$

The two predictions for the wavelength of Mercury's green line obtained by treating double-slit interference and single-slit diffraction individually were:

$$\lambda = (5.9 \pm 0.5) \times 10^{-7}\text{m},$$

$$\lambda = (5.3 \pm 0.1) \times 10^{-7}\text{m}.$$

Using these two predictions and their respective uncertainties, we report our final result to be:

$$\lambda = (5.3 \pm 0.3) \times 10^{-7}\text{m}.$$

This is a 2.97% error away from the true value of 5.462×10^{-7}m, so our experiment exhibits a high degree of accuracy. The true value, however, falls outside the uncertainty range of our experimental value obtained from treating just the single-slit diffraction, suggesting the presence of sources of systematic error in that part of the experiment. A couple of possible sources of systematic error include the difficulty to identify successive diffraction minima due to the double-slit interference pattern, and the misalignment of the light source, filter, diverging lens, collimator, the double-slit, or the eyepiece.

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