Faraday Rotation

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This paper reports the results of analyzing the Faraday rotation effect of SF-59 glass subject to a magnetic field generated by current through a solenoid. We verify Malus' Law to determine the orientation of the first polarizer. We set the nominal angle at an optimal $\phi = 45°$ to make relative intensity measurements at various magnetic field strengths. The relative intensity is measured using a photo diode and a simple circuit that allows for voltage readout. The exact magnetic field strength generated by a current through a solenoid of finite length is calculated and used to compute magnetic field strengths at various currents. A plot of the magnetic field strength integrated over the length of the optical material against the Faraday rotation angle is produced and a linear fit is obtained. The experimental value of the Verdet constant was obtained from the slope of the linear fit, and compared to two theoretical values: one provided by the manufacturer and one predicted by the dispersion relation.

I. INTRODUCTION

Faraday rotation is a magneto-optic effect that causes the polarization vector of light to rotate as it passes through an optical material in an external magnetic field [1]. First discovered by Michael Faraday in 1845, it was the first experimental evidence linking light to magnetic phenomena [2]. As the magnetic field is applied, the material used for the experiment exhibits circular birefringence, the splitting of the material's refractive index into distinct values for right and left circularly polarized light [3].

Since right and left circularly polarized light form a basis for polarization states, the initial linearly polarized state can be expressed as a linear combination of the right and left circularly polarized states. Now, as the light passes through the optical material, the two circularly polarized states propagate at different rates due to the difference in refractive indices, causing one circular polarization state to pick up a phase shift relative to the other [3]. As we translate our superposition of circular polarization states after it has passed through the material back to linear polarization, we notice that the polarization angle has rotated by a certain angle θ with respect to our initial polarization due to the relative phase shift.

The equation that governs the Faraday rotation effect is:

$$\theta = \mathcal{V}BL = \frac{\pi \Delta n L}{\lambda}, \tag{1}$$

where $\mathcal{V}$ is the Verdet constant which depends on the optical material used, B is the magnetic field strength, L is the length of the material, and ΔN is the difference in refractive indices for right and left circularly polarized light, and λ is the wavelength of light passing through the material [3].

In this experiment, we will pass monochromatic light of known polarization state through an optical material in a magnetic field to obtain an experimental value of the Verdet constant. The magnetic field is generated by a solenoid, and we control the magnetic field strength by controlling the current passing through the solenoid. The laser passes through an initial polarizer to set the light to a particular linear polarization, then goes through the optical material in the magnetic field, and passes through a second polarizer before the relative intensity is read using a photodiode.

II. PROCEDURE

II.1. Verifying Malus' Law

Before we carry out actual measurements to quantify the Faraday effect and obtain an experimental value of the Verdet constant, we must identify the initial polarization state of the ingoing monochromatic light. We do this by rotating the second polarizer through 360° and verifying Malus' Law:

$$\mathcal{I} = \mathcal{I}_0 \cos^2 \phi, \tag{2}$$

where $\mathcal{I}_0$ is the ingoing light intensity, $\mathcal{I}$ is the outgoing light intensity, and ϕ is the angle between the ingoing polarization state and the polarization angle of the polarizer [4].

II.2. Verdet Constant and the Optical Material: SF-59

The Verdet constant is a material-dependent and wavelength-dependent constant that describes the strength of the Faraday rotation. It follows the dispersion relation given by:

$$\mathcal{V} = \frac{\pi}{\lambda}\left(a + \frac{b}{\lambda^2 - \lambda_0^2}\right). \tag{3}$$

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[3]

The particular optical material used for this experiment is SF-59 glass, for which we know the constants a, b, and λ_0 as follows:

$$a = 2843.20 \times 10^{-9}\ \mathrm{T}^{-1},$$

$$b = 112.5192 \times 10^{-20}\ \mathrm{m}^2/\mathrm{T}, \text{ and}$$

$$\lambda_0 = 175.3\ \mathrm{nm}$$

According to our dispersion relation, at $\lambda = 650$nm we should have:

$$\mathcal{V} \approx 27.6\mathrm{rad/Tm}.$$

However, the manufacturer claims that the optical material has a Verdet constant of 23 rad/Tm at a wavelength of 650nm [5]. We will compare our experimental value to both of these theoretical values to evaluate the accuracy of our experiment.

II.3. Measuring Intensity with the Photodiode

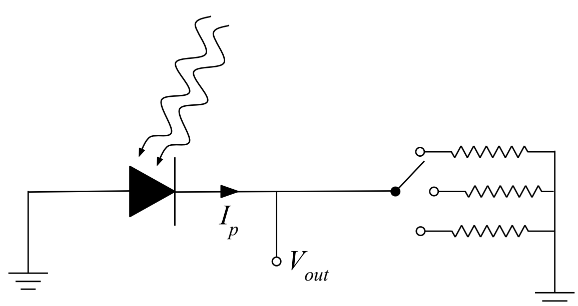

FIG. 1. This is a schematic diagram of a circuit used to measure incident light intensity using a photodiode. The switch allows the use of three different resistors to adjust the amplification of the photocurrent to the voltage output.

A photodiode is an electronic device the converts incident light intensity into a proportional current signal [3]. There is some efficiency factor, a proportionality constant, that relates the intensity of light incident and the output photocurrent, so we may write:

$$I_p = \epsilon \mathcal{I}, \tag{4}$$

where I_p is the output photocurrent, ϵ is the efficiency factor, and $\mathcal{I}$ is the incident light intensity. In general, the efficiency factor ϵ depends on the photodiode and the laser orientation, but since these are unchanged throughout our experiment, we can treat the efficiency factor as a constant. Since it is generally more convenient to perform signal readout for voltage output rather than current output, we set up a simple circuit schematically shown in Figure 1. To avoid reverse bias, we keep the reverse bias voltage to well below 300mV as recommended by the manufacturer, and switching between three resistors allows us to accommodate this restriction. The three different resistor settings are: 1kΩ, 3kΩ, and 10kΩ. The photocurrent and the voltage output are related by Ohm's Law [6]:

$$V_{\mathrm{out}} = I_p R. \tag{5}$$

From Equations 4 and 5, we obtain:

$$V_{\mathrm{out}} = \epsilon R \mathcal{I} \tag{6}$$

Thus, in terms of relative intensities, we have:

$$\frac{V_{\mathrm{out}}}{V_0} = \frac{\mathcal{I}}{\mathcal{I}_0}, \tag{7}$$

where $\mathcal{I}_0$ is the maximum possible intensity of incident light on the diode when there is no magnetic field and the second polarizer is aligned with the first polarizer, and V_0 is the corresponding voltage output. The efficiency factor ϵ and the resistance R cancel out, provided that we use the same resistor for the data set. Therefore, we can measure the relative intensities by measuring relative voltage outputs.

II.4. The Magnetic Field and Calculating the Faraday Rotation

We can calculate the magnetic field strength as a function of the position along the axis of the solenoid of finite length by applying the Biot-Savart Law. Let us set up the coordinate system such that the x-axis is aligned with the central axis of the solenoid and the origin is located at exactly half-way through the solenoid. Then,

$$\begin{aligned} B_x &= \frac{\mu_0 N I}{2} R^2 \int_{-L/2}^{L/2} \frac{dx'}{[(x - x')^2 + R^2]^{3/2}} \\ &= \frac{\mu_0 N I}{2} \left(\frac{x + \frac{L}{2}}{\sqrt{(x + \frac{L}{2})^2 + R^2}} - \frac{x - \frac{L}{2}}{\sqrt{(x - \frac{L}{2})^2 + R^2}} \right). \end{aligned} \tag{8}$$

[7]

For our system, we know that the length of the solenoid L is 150mm and the radius R is 17.5mm [3]. We can make a further simplification since the manufacturer claims that the field strength can be calculated by 11.1mT/A at the center of the solenoid [5]. From our equation, at $x = 0$, we have:

$$B_0 = \frac{\mu_0 NI}{2}\left(\frac{L}{\sqrt{\frac{L^2}{4}+R^2}}\right)$$
$$\implies \frac{\mu_0 NI}{2} = \left(\frac{\sqrt{\frac{L^2}{4}+R^2}}{L}\right) B_0 \tag{9}$$
$$\implies \frac{\mu_0 NI}{2} = \left(\frac{\sqrt{\frac{L^2}{4}+R^2}}{L}\right) (11.1\text{mT/A})I$$

Hence, we obtain:

$$B_x = (11.1\text{mT/A})\left(\frac{\sqrt{\frac{L^2}{4}+R^2}}{L}\right)\left(\frac{x+\frac{L}{2}}{\sqrt{(x+\frac{L}{2})^2+R^2}} - \frac{x-\frac{L}{2}}{\sqrt{(x-\frac{L}{2})^2+R^2}}\right) I \tag{10}$$

Our first equality in Equation 1 assumes a uniform magnetic field strength along the material, but for a current through a solenoid of finite length, this is not the case as we have shown. For a solenoid along the x-axis and a magnetic field strength that depends on the position along the axis, we can infer from Equation 1 that:

$$d\theta = \mathcal{V}Bdx$$
$$\implies \theta = \mathcal{V}\int_\ell Bdx, \tag{11}$$

where the integral is taken over the region where the optical material lies [5].

From Equations 10 and 11 we obtain:

$$\theta = \mathcal{V}I(11.1\text{mT/A})\left(\frac{\sqrt{\frac{L^2}{4}+R^2}}{L}\right)\int_\ell\left(\frac{x+\frac{L}{2}}{\sqrt{(x+\frac{L}{2})^2+R^2}} - \frac{x-\frac{L}{2}}{\sqrt{(x-\frac{L}{2})^2+R^2}}\right) dx. \tag{12}$$

Finally, we can evaluate the integral by using the fact that the optical material is 10.0cm long [3] and substitute in constants we know to yield:

$$\theta \approx (0.00109)\mathcal{V}I \tag{13}$$

III. RESULTS

III.1. Malus' Law

The second polarizer was rotated through 360° at 10° increments. The second polarizer's polarization angle in degrees is plotted against the output voltage in Figure 2. In addition to the 10°-incremented measurements, we directly measured the maxima and minima to obtain the following results:

$$V_{\text{max},1} = 0.1519\text{V, when } \theta_{\text{max},1} = (65 \pm 2.5)^\circ,$$

$$V_{\text{max},2} = 0.1518\text{V, when } \theta_{\text{max},2} = (244 \pm 2.5)^\circ,$$

$$V_{\text{min},1} = 0.0002\text{V, when } \theta_{\text{min},1} = (335 \pm 2.5)^\circ, \text{ and}$$

$$V_{\text{min},2} = 0.0002\text{V, when } \theta_{\text{min},2} = (156 \pm 2.5)^\circ.$$

We will use these angle measurements that give maximum and minimum voltage output to determine the nominal angle ϕ. Specifically, we will say that for our experimental purposes, $\phi = 0$ when the second polarizer is set to approximately $\theta_{\text{max},1} = 65^\circ$.

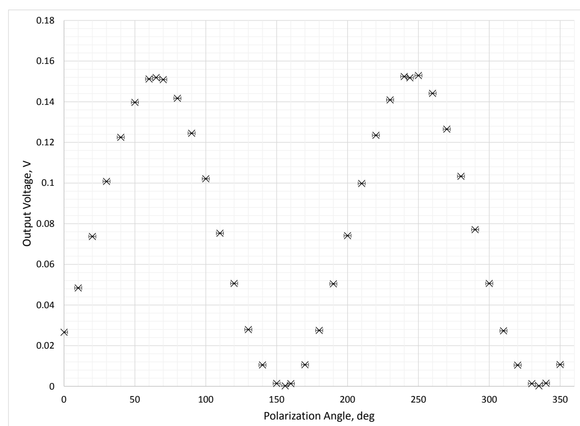

FIG. 2. This is a plot of the second polarizer's polarization angle in degrees against the voltage output obtained for a resistor setting of $R = 1\text{k}\Omega$.

III.2. Current through Solenoid and Relative Intensity Measurements for $\phi = 90^\circ$

According to our Malus' Law data, we adjust the second polarizer to an angle of 335° so that it makes a nominal angle $\phi = 90^\circ$ with the initial polarization state of the incident light. For this orientation, we measured the voltage output from the photodiode for different currents passed through the solenoid. Given a nominal angle ϕ when no current is passing through the current, Faraday rotation will alter the difference between the polarization state of the light incident on the second polarizer and the polarization angle to $\phi \pm \theta$, depending on the direction of the Faraday rotation. Thus, by Equation 2, we have:

$$\mathcal{I} = \mathcal{I}_0 \cos^2(\phi \pm \theta) \tag{14}$$

Combining Equations 7 and 14, we obtain:

$$\begin{aligned} \cos^2(\phi \pm \theta) &= \frac{V}{V_0} \\ \implies \phi \pm \theta &= \cos^{-1}\left(\sqrt{\frac{V}{V_0}}\right) \\ \implies \theta &= \pm\left(\cos^{-1}\left(\sqrt{\frac{V}{V_0}}\right) - \phi\right) \end{aligned}$$

We calculate this Faraday rotation angle for every voltage output measurement.

Using Equations 11 and 13, we see that:

$$\int_\ell B dx \approx (0.00109) I. \tag{15}$$

We plot $\int_\ell B dx$ against the Faraday rotation angle θ and perform a linear fit so that the gradient of the linear fit gives us an experimental value of our Verdet constant $\mathcal{V}$ following Equation 11. This plot is shown in Figure 3.

Our data shows a clear linear trend except when the magnetic field strength, and thus the Faraday rotation, are small. This systematic error at the lower end of our data set is most likely due to the fact that our expression for the magnetic field strength in Equation 10 is not accurate when the current is relatively low. Even small discrepancies in the solenoid such as with the material used and the orientation of each layer and turns, cause relatively large deviations of the true magnetic field strength from our predicted magnetic field strength. Keeping this source of systematic error in mind, we rejected the first seven data points that deviate from the linear trend when computing the linear fit. The Verdet constant for this trial is:

$$\mathcal{V} = (20.38 \pm 0.07) \text{ rad/Tm}.$$

III.3. Current through Solenoid and Relative Intensity Measurements for $\phi = 45°$

From Malus' Law in Equation 2, we see that:

$$\left|\frac{\partial \mathcal{I}}{\partial \phi}\right| = 2\mathcal{I}_0 \cos\phi \sin\phi,$$

and this reaches a maximum value at $\phi = 45°$, which means that we expect the largest response of relative intensity at this nominal output. This is ideal, since the Faraday rotation angle is small, and we would like to be able to see a more clear change in relative intensity as we change the magnetic field strength.

According to our Malus' Law data, we adjust the second polarizer to an angle of 20° so that it makes a nominal angle $\phi = 45°$ with the initial polarization state of the incident light. We measure the voltage output from the photodiode for different currents passed through the solenoid, and follow the same calculations as when we set $\phi = 90°$.

We once again plot $\int_\ell B dx$ against the Faraday rotation angle θ and perform a linear fit as shown in Figure 4.

Our data once again shows significant systematic error which we attribute to discrepancies between the true magnetic field strength and our expression in Equation 10. Since the slope seems to increase slightly with increased magnetic field strength, we expect that the solenoid produces stronger than predicted magnetic field at low currents and weaker than expected magnetic field at high currents. We did not reject any data points for this trial since nonlinear tendencies are seen at both ends of the data set, but we would expect the linear fit should still give us a reasonable experimental value of the Verdet constant. Indeed, the Verdet constant for this trial is:

$$\mathcal{V} = (23.8 \pm 0.3) \text{ rad/Tm}.$$

Since $\phi = 45°$ is the optimal nominal angle at which we can make measurements, we take this experimental value to be our final result.

IV. ERROR ANALYSIS

IV.1. Uncertainty Propagation

We show a sample uncertainty propagation calculation here for the first datum in the $\phi = 90°$ trial where the current through the solenoid is $I = (0.004 \pm 0.001)$A and the output voltage is $V = (0.0011 \pm 0.0001)$V. The voltage output corresponding to the initial light intensity for this trial was $V_0 = (1.519 \pm 0.0001)$V.

We calculate the magnetic field strength integrated over the length of the material according to Equation 15. The factor 0.00109 is a constant, so the uncertainty in the integrated magnetic field strength is simply the uncertainty in the current multiplied by this constant. Thus,

$$\begin{aligned} \alpha\left(\int_\ell B dx\right) &= (0.00109)\alpha_I \\ &= 1.09 \times 10^{-6} \text{ Tm}. \end{aligned}$$

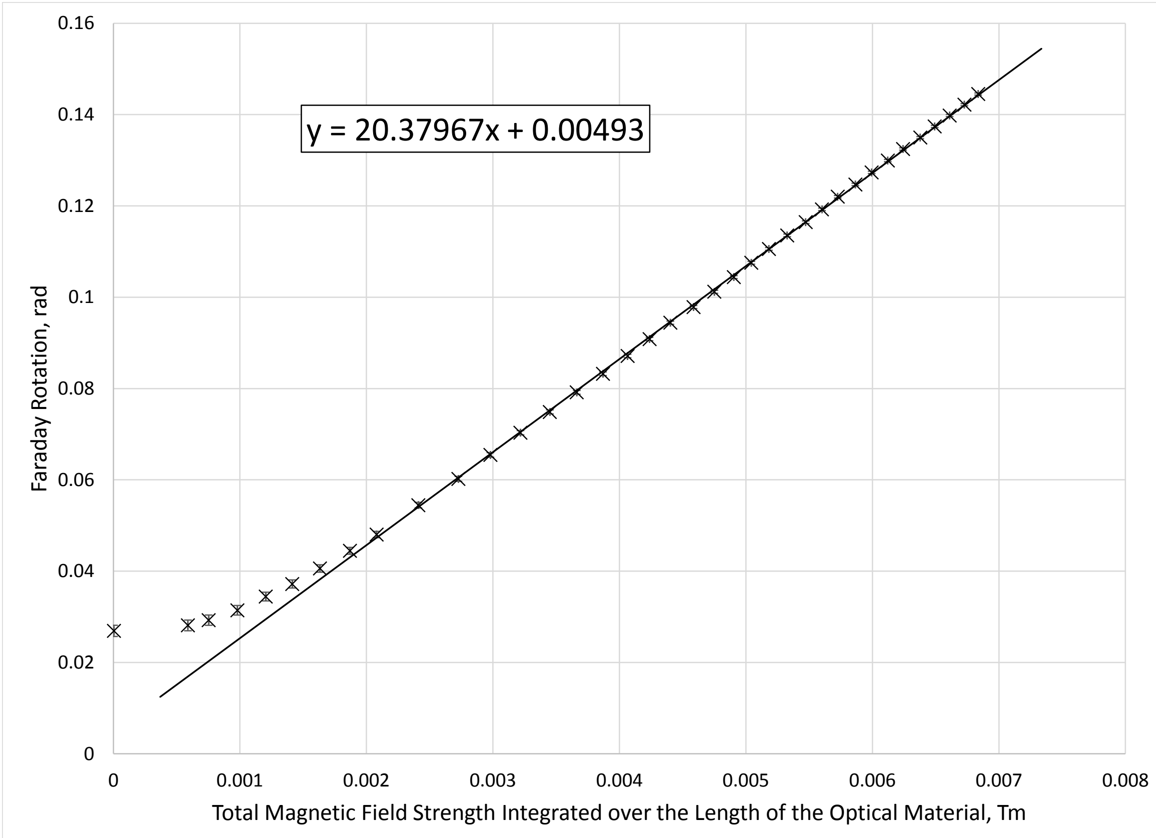

FIG. 3. This is a plot of the magnetic field strength integrated over the length of the optical material against the Faraday rotation angle for $\phi = 90°$. A linear fit to the data is also plotted and its equation displayed.

This is the size of the horizontal error bars in Figures 3 and 4, but they are too small to be seen.

The relative intensity $\mathcal{I}/\mathcal{I}_0$ for this datum is:

$$\begin{aligned}\frac{\mathcal{I}}{\mathcal{I}_0} &= \frac{V}{V_0} \\ &= \frac{0.0011\text{V}}{1.519\text{V}} \\ &\approx 0.00072.\end{aligned}$$

The uncertainty in this measurement is calculated by [8]:

$$\begin{aligned}\alpha(\mathcal{I}/\mathcal{I}_0) &= (\mathcal{I}/\mathcal{I}_0)\sqrt{\left(\frac{\alpha_V}{V}\right)^2 + \left(\frac{\alpha_{V_0}}{V_0}\right)^2} \\ &\approx 0.00007.\end{aligned}$$

Finally, the angle $\phi \pm \theta$ is calculated by:

$$\phi \pm \theta = \cos^{-1}\left(\sqrt{\frac{V}{V_0}}\right),$$

so we will utilize the calculus method to approximate our uncertainty propagation [8]. First, let $u = \sqrt{\mathcal{I}/\mathcal{I}_0}$. Then,

$$\begin{aligned}\alpha_u &= \left|\frac{1}{2}\left(\frac{\mathcal{I}}{\mathcal{I}_0}\right)^{-1/2}\right|\alpha(\mathcal{I}/\mathcal{I}_0) \\ &= \frac{\alpha(\mathcal{I}/\mathcal{I}_0)}{2\sqrt{\mathcal{I}/\mathcal{I}_0}}.\end{aligned}$$

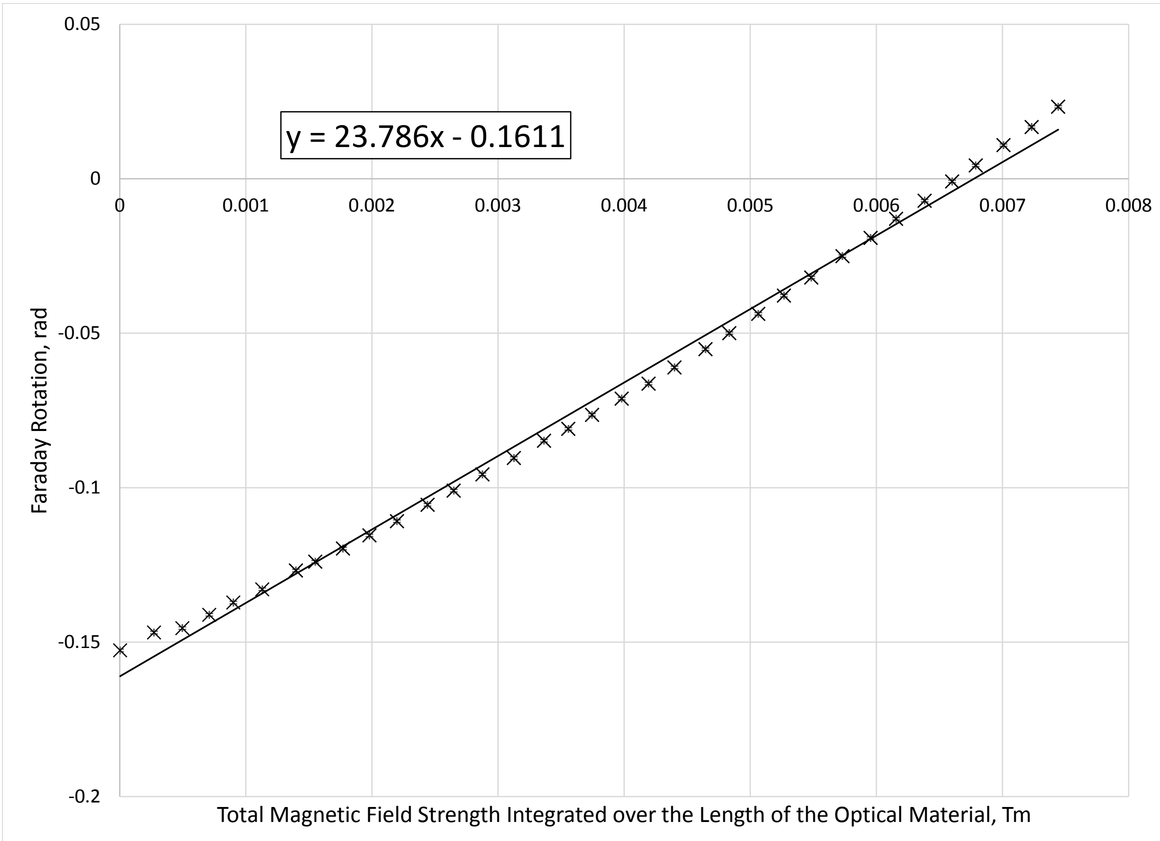

FIG. 4. This is a plot of the magnetic field strength integrated over the length of the optical material against the Faraday rotation angle for $\phi = 45°$. A linear fit to the data is also plotted and its equation displayed.

Now, since $\phi \pm \theta = \cos^{-1}(u)$,

$$\begin{aligned}\alpha(\phi \pm \theta)^2 &= \left(\frac{\partial(\phi \pm \theta)}{\partial u}\right)^2 (\alpha_u)^2 \\ &= \frac{1}{1-u^2}\left(\frac{\alpha(\mathcal{I}/\mathcal{I}_0)}{2\sqrt{\mathcal{I}/\mathcal{I}_0}}\right)^2 \\ &= \frac{(\alpha(\mathcal{I}/\mathcal{I}_0))^2}{4(1-\mathcal{I}/\mathcal{I}_0)(\mathcal{I}/\mathcal{I}_0)} \\ &\approx 0.001 \text{ rad.}\end{aligned}$$

Since our nominal angle ϕ is constant, we take this as our uncertainty for our Faraday rotation angle:

$$\alpha_\theta \approx 0.001 \text{ rad.}$$

This is shown as vertical error bars in our Figures 3 and 4, although the uncertainty is negligible for most data points.

IV.2. Linear Regression

Following Hughes & Hase, we calculate the uncertainty in our slope of linear fit, and thus the uncertainty of our experimental values of the Verdet constant using:

$$\alpha_{CU} = \sqrt{\frac{1}{N-2}\sum_i (y_i - mx_i - c)^2}, \text{ and}$$

$$\alpha_m = \alpha_{CU}\sqrt{\frac{N}{\Delta}},$$

where $\Delta = N\sum_i x_i^2 - \left(\sum_i x_i\right)^2$ [8].

IV.3. Percentage Error and Accuracy

Comparing our experimental value of the Verdet constant to the manufacturer's reporting, we obtain a percentage error of:

$$\begin{aligned}\%_e &= \frac{\theta_{\text{exp}} - \theta_{\text{the}}}{\theta_{\text{the}}} \times 100\% \\ &= \frac{23.8 \text{ rad/Tm} - 23 \text{ rad/Tm}}{23 \text{ rad/Tm}} \times 100\% \\ &= 3.48\%.\end{aligned}$$

Comparing our experimental value to the theoretical value predicted by the dispersion relation, we get:

$$\begin{aligned}\%_e &= \frac{\theta_{\text{exp}} - \theta_{\text{the}}}{\theta_{\text{the}}} \times 100\% \\ &= \frac{23.8 \text{ rad/Tm} - 27.6 \text{ rad/Tm}}{27.6 \text{ rad/Tm}} \times 100\% \\ &= -13.77\%.\end{aligned}$$

Therefore, our experimental value exhibits a reasonably degree of accuracy in both cases, but our experiment seems to suggest that the manufacturer is closer to reporting the true value of the Verdet constant than the one predicted by the dispersion relationship. Both of the theoretical values of the Verdet constant fall outside of our uncertainty range, which suggests sources of systematic error.

V. CONCLUSION

In this experiment, we successfully quantified the Faraday rotation effect and generated plots of the magnetic field integrated over the length of the material against the Faraday rotation angle to obtain an experimental value of the Verdet constant from the slope of the linear fit to a reasonable degree of accuracy. We first verified Malus' Law by rotating the second polarizer through 360° so that we can set up an optimal nominal angle of $\phi = 45°$ to make Faraday rotation measurements. We obtained two theoretical values of the Verdet constant: one from the manufacturer's reporting and the other from the dispersion relation. We measured relative intensities by using a photodiode and a simple circuit to perform voltage output readout. Finally, we found the magnetic field strength generated by a current through a solenoid of finite length to obtain a relationship between current and magnetic field strength. For the optimal nominal angle $\phi = 45°$, we obtained an experimental value of the Verdet constant:

$$\mathcal{V} = (23.8 \pm 0.3)\text{rad/Tm}.$$

This is a 3.48% error away from the manufacturer-reported value and -13.77% away from the value predicted by the dispersion relation, so our experiment rules in favor of the manufacturer reporting. However, both theoretical values fall outside the uncertainty range of our experimental value, suggesting the presence of sources of systematic error. A few possible sources of systematic error include the discrepancy between predicted and actual magnetic field strength generated by the current through the solenoid, misalignment of the laser, polarizer, or photo diode, and the misalignment of the optical material within the solenoid.

Appendix A: Raw Data for Malus' Law Verification

Polarization Angle, °	Output Voltage, V
0	0.0266
10	0.0484
20	0.0737
30	0.1008
40	0.1225
50	0.1397
60	0.1512
70	0.1509
80	0.1417
90	0.1245
100	0.1021
110	0.0753
120	0.0506
130	0.0279
140	0.0105
150	0.0014
160	0.0013
170	0.0106
180	0.0275
190	0.0504
200	0.0741
210	0.0998
220	0.1235
230	0.1409
240	0.1524
250	0.1529
260	0.1441
270	0.1265
280	0.1033
290	0.0771
300	0.0506
310	0.0273
320	0.0104
330	0.0013
340	0.0015
350	0.0107

TABLE I. Polarization angle of the second polarizer and the output voltage

Appendix B: Raw Data for $\phi = 90°$

Current through the Solenoid, A	Output Voltage, V
0.004	0.0011
0.540	0.0012
0.690	0.0013
0.898	0.0015
1.104	0.0018
1.296	0.0021
1.497	0.0025
1.714	0.0030
1.907	0.0035
2.210	0.0045
2.500	0.0055
2.732	0.0065
2.950	0.0075
3.163	0.0085
3.358	0.0095
3.548	0.0105
3.726	0.0115
3.885	0.0125
4.036	0.0135
4.204	0.0145
4.355	0.0155
4.496	0.0165
4.623	0.0175
4.750	0.0185
4.883	0.0195
5.017	0.0205
5.135	0.0215
5.249	0.0225
5.377	0.0235
5.495	0.0245
5.612	0.0255
5.722	0.0265
5.847	0.0275
5.952	0.0285
6.060	0.0295
6.167	0.0305
6.267	0.0315

TABLE II. The current through the solenoid and the voltage output for $\phi = 90°$

Appendix C: Raw Data for $\phi = 45°$

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Current through the Solenoid, A	Output Voltage, V
0.002	0.1884
0.249	0.1868
0.455	0.1864
0.651	0.1852
0.826	0.1841
1.036	0.1829
1.281	0.1812
1.421	0.1804
1.623	0.1792
1.815	0.1780
2.015	0.1767
2.237	0.1752
2.428	0.1739
2.636	0.1724
2.865	0.1709
3.084	0.1693
3.261	0.1682
3.433	0.1669
3.649	0.1654
3.844	0.1640
4.033	0.1625
4.258	0.1608
4.431	0.1593
4.642	0.1575
4.828	0.1558
5.027	0.1541
5.253	0.1521
5.458	0.1504
5.644	0.1486
5.851	0.1469
6.051	0.1451
6.222	0.1436
6.424	0.1417
6.628	0.1400
6.822	0.1381

TABLE III. The current through the solenoid and the voltage output for $\phi = 45°$

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[7] The University of Rhode Island. "Magnetic Field on the Axis of a Solenoid." *Magnetic Field on the Axis of a Solenoid*, 26 Mar. 2008, www.phys.uri.edu/gerhard/PHY204/tsl215.pdf.

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