How to Solve Comparison Problems
ASTR 1010: Introductory Astronomy 1

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Abstract

Often times in astronomy classes (along with many other situations), we are asked to determine how one quantity changes (the semi-major axis of a planet's orbit, for example) when we change something that it depends on (the period of a planet's orbit, to continue the example). Knowing how to solve these problems is extremely valuable, if for no other reason than to make your time in this class easier. Let us work through an example which you have already seen in the *Kepler's Laws* lab in Week 3 as a reference guide for problems of this type that you may encounter in the future.

First, we remind ourselves that Kepler's third law, which relates the semi-major axis of a planet's orbit (*basically* how far that planet is from the Sun) to that planet's orbital period (how long it takes the planet to go around the Sun once) is

$$P^2 = a^3$$

where P is the period in **years** and a is the semi-major axis in **Astronomical Units (AU)**. Make sure your units are correct, otherwise this formula fails!

We are told the following (Kepler's Laws III.1, p. 51): "Rearrange the equation for Kepler's 3rd Law to give an expression for the value of the semi-major axis, a, in terms of a given period, P (i.e. solve for a). If the period increases by a factor of two, how much does the semi-major axis change by?"

The first step is to isolate a on one side of the equation. Algebraically speaking this is a simple task, but may not be obvious if you haven't done this kind of math for a long time. Starting with $P^2 = a^3$ we can take the *cube root* of both sides to "un-cube" a:

$$\sqrt[3]{P^2} = \sqrt[3]{a^3} \rightarrow \boxed{a = \sqrt[3]{P^2} = P^{\frac{2}{3}}}$$

Now a is by itself. Nice! Notice that I changed $\sqrt[3]{P^2}$ into $P^{\frac{2}{3}}$. This is just another way to write the same thing, but makes everything look nicer.

Now we are tasked with answering the question "what happens to a when we increase P by a factor of two?" First of all, anytime you see the words "increase by a factor of" think "multiply by" (the same goes for "decrease by a factor of" which really means "divide by"). So the question becomes, "what happens to a when we multiply P by two?" To find out, let us set up two different scenarios: old and new. The old scenario is what we will call everything *before* we double the period, and the new scenario is what we will call everything *after* we double the period.

So we have, $a_{\text{old}} = (P_{\text{old}})^{\frac{2}{3}}$ and $a_{\text{new}} = (P_{\text{new}})^{\frac{2}{3}}$. From the problem, we know that

$$P_{\text{new}} = 2P_{\text{old}}$$

Thus

$$a_{\text{new}} = (2P_{\text{old}})^{\frac{2}{3}}$$

All I've done here is substitute $2P_{\text{old}}$ in for P_{new}. We're getting very close to being done now. All we need to do is *compare* a_{new} to a_{old} (hint: anytime you see "compare" or something like it, think fractions!):

$$\frac{a_{\text{new}}}{a_{\text{old}}} = \frac{(2P_{\text{old}})^{\frac{2}{3}}}{(P_{\text{old}})^{\frac{2}{3}}} = \left(\frac{2P_{\text{old}}}{P_{\text{old}}}\right)^{\frac{2}{3}} = 2^{\frac{2}{3}}\left(\frac{P_{\text{old}}}{P_{\text{old}}}\right)^{\frac{2}{3}} = 2^{\frac{2}{3}}$$

Notice that we were able to pull the $\frac{2}{3}$ exponent out of the fraction so that we could cancel the P_{old}'s. Now, all we have to do is multiply both sides by a_{old} and we are done!

$$a_{\text{new}} = 2^{\frac{2}{3}} a_{\text{old}}$$

What this all means is that if we double the period of planet in orbit around the Sun then its new semi-major axis will be $2^{\frac{2}{3}}$ times larger than its original semi-major axis before we doubled the period.

This all looks like a lot of math (which it is!) but don't be discouraged. We will get very good at these kinds of comparisons as we do more and more of them. The general strategy is this:

1. Isolate the variable that you are investigating (a in our case, since the question asks how a changes)
2. Create two scenarios: one where you've changed nothing (i.e. use the original formula) and one which includes the changes you are investigating
3. *Compare* the new scenario to the old one by *taking a ratio*: $\frac{\text{new scenario}}{\text{old scenario}}$
4. Plug in and simplify

Best of luck, and keep looking up!