1 Homework for 11.4: Tensor Product of Two Vectors

Due 12/05/2017

1.

Let $x = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$, $y = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$, and $v = \begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix}$. Compute the outer products $x \otimes y$ and $y \otimes x$, and verify that $x \otimes y$ indeed satisfies the definition of a tensor product between vectors i.e. $(x \otimes y)(v) = x(v, y)_Y$.

2.

Show that Definition 11.4.1 is a definition specific to real vectors that does not generalize to complex vectors with the usual inner product. In other words, come up with a counter example of complex vectors for which the outer product for complex vectors (discussed in class) does not satisfy Definition 11.4.1.

3.

a) Denote by $\{\hat{x}_1, \hat{x}_2\}$ the standard basis in $\mathbb{R}^2$ and $\{\hat{y}_1, \hat{y}_2, \hat{y}_3\}$ the standard basis in $\mathbb{R}^3$. Write down a basis, β, for $\mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ using tensor products of the standard basis vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$.

b) Let $v = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $w = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}$. Compute the tensor product $v \otimes w$ in terms of the basis vectors in β.

4.

Prove that the tensor product is bilinear. i.e.

For $a, b, c, d \in \mathbb{R}$, $v_1, v_2, x \in V$, and $w_1, w_2, y \in W$, show the following two properties:

$$(av_1 + bv_2) \otimes y = av_1 \otimes y + bv_2 \otimes y$$

$$x \otimes (cw_1 + dw_2) = cx \otimes w_1 + dx \otimes w_2$$